Single Value Decomposition to Maximize the Signal-to-Noise Ratio on Digital Image

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Abstract. One of the most serious problem found by users to analyze digital image is due the strong presence of *salt and pepper* noise which frequently masks the information of interest. Noise corrupts the signal and makes information extraction more difficult. This work brings a study of the single values matrix obtained from the Eigen Values-Eigen Vectors Decomposition of digital image. The aim of it is to maximize the signal-to-noise ratio and get a better unsupervised classification. Understanding the effects of the single values matrix over all pixels, it is proposed a denoising in order to get better results on unsupervised classifications using Self Organizing Maps. To compare the performance of this filter, it is presented the classifications of the original and filtering images. In the Recognition and in the Facial Identification, however this kind of problem and common. However the technical combination of the search minimize this effect.

1. Introduction

To maximize the signal-to-noise ratio as a form to remove the *salt and pepper* from the images, in this study was tested the single values decomposition [1]. Unfortunately it is difficult to define a modification criteria in the single values that reduces the noise in the digital image and, at the same time, doesn't make the filtered image loses much of the features that characterize objects in the real scene. To avoid this problem the authors have studied some relations between the characteristics of the single values on the formation of the pixels and the characteristics of *salt and pepper* noise formation in this kind of images [2,3]. The reconstruction of images from individual single values permits a good approximation to the original image without the smallest pixels oscillations of a same scene. These oscillations difficult the interpretation of structures in the image as soon as your classification. To evaluate the performance of this criteria of denoise, both original and filtered images were classified using the Self-Organizing Map (SOM) unsupervised classifier. Self-Organizing Map (SOM) [4,5] is an algorithm that has the special property of effectively creating spatially organized internal representations. And it is very useful as an unsupervised method of digital image classification [6]. Scientific reference of salt and pepper is found since last four decades. The salt and pepper is a typical feature easily recognized in digital images registered in low resolution camera by it appearance of "salt and pepper", i.e., as a profusion of small white and black points, sometimes concentrated in parts of the images, sometimes spread over the whole image. This kind of noise is present in digital images as a consequence of data processing. Most of the tentatives to eliminate or to reduce the salt and pepper [3] leads to a degradation of the spatial resolution of the images, reducing the chances to discriminate the objects or targets present in the image and so, causing bad results when images are automatically classified.

2. Image decomposition

The equation 1 brings the reconstruction of a image **I** from its eigen values **S** and its eigen vectors **U**, **V**. This is known as single values decomposition because the values **Si** of the matrix **S** are the positive square roots of the eigen values of $\mathbf{I}^T.\mathbf{I}$ or $\mathbf{I}.\mathbf{I}^T$. So **Si** are designed single values of **I** [1].

$$\mathbf{I}=\mathbf{U}.\mathbf{S}.\mathbf{V}^T \qquad (1)$$

Considering the image decomposition case, the importance of this decomposition is in its single values matrix **S**. Due your analysis is possible to verify the contribution degree of these values (**Si**) on the process of formation of all image pixels. Now we can verify the reconstruction process of an image **I** of dimension NxN, from the matrixes $\mathbf{U}_{(NxN)}$, $\mathbf{V}_{(NxN)}$ and the diagonal matrix $\mathbf{S}_{(NxN)}$ (2).

$$\mathbf{I}_{NxN} = \mathbf{U}_{NxN} \cdot \begin{bmatrix} S_{11} & 0 \\ 0 & S_{NN} \end{bmatrix}$$

We can take a set of reconstructed images $\mathbf{I}^i_{NxN}$ (3), with 1 <= i <= N. By the sum of this reconstructed images, that use a **S** partial, we take another way of reconstruct the original image (4).

$$\mathbf{I}^i_{NxN} = \mathbf{U}_{NxN} \cdot \begin{bmatrix} S_{11} & 0 \\ 0 & S_{NN} \end{bmatrix} \cdot \mathbf{V}_{NxN}^T \ \{ S_{KK} = 0 \text{ if } K \neq i \} \qquad (3)$$

$$\mathbf{I}_{NxN} = \sum_{i=1}^{N} \mathbf{I}^i_{NxN} \qquad (4)$$

As, in the reconstruct process (3) it was taken a maximization of a unique single value, each image $\mathbf{I}^i$ **NxN**, of the set, carries out the representatively of its respectively single value in all pixels of the original image $\mathbf{I}_{NxN}$.

3. Denoise criteria

Now, considering this data (Fig. 1), i.e., the original image $\mathbf{I}_{100x100}$, and taking the 100 decompositions images $\mathbf{I}^i_{100x100}$ (1 <= i <= 100) that represents the contribution of each single value of $\mathbf{I}_{100x100}$ on the formation of their pixels (3), it is easy to verify the different degree of contribution of the single values in the image space (the single values are taken in a inverse order of their magnitudes). With some example of the reconstructed images are easy to see that each single value has a part of contribution in all original pixels, but with a particular way (Fig. 2-11). Observing the reconstruction process of the original image from these decompositions images (4) it is easy to see that each original pixel $\mathbf{I}_{(i,j)}$ obeys the same reconstruction process of its particular position (5).

$$I_{(i,j)} = \sum_{i=1}^{N} I^{i}_{(i,j)} \qquad (5)$$

Figure 1. Original image $\mathbf{I}_{100x100}$.

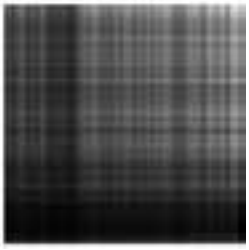

Figure 2. The reconstructed image $\mathbf{I}^1_{100x100}$.

Figure 3. There constructed image $\mathbf{I}^2_{100x100}$.

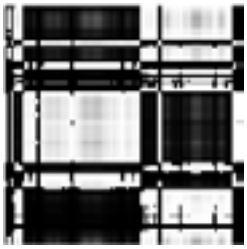

Figure 4. The reconstructed image $\mathbf{I}^3_{100x100}$.

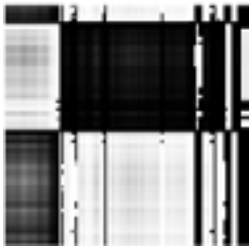

Figure 5. The reconstructed image $\mathbf{I}^4_{100x100}$.

Each pixel position has different values in each reconstructed image. This feature leads to a study of the reconstructed pixels that have greater approximations of the original pixel (Figure 12-22).

(6)

A reconstructed image which has its pixels values reconstructed with only its approximation, results on a filtered image where, each pixel, has an emphasized contribution of the single value that had more contribution in the primary decomposition process of the original image (7).

$$\sum_{i=1}^{N}\sum_{j=1}^{N} I'_{(i,j)} = I^{k}_{(i,j)} \quad \text{, where } D^{k}_{(i,j)} = \min(D_{(i,j)})$$

Figure 6. The reconstructed image $\mathbf{I}^{10}_{100x100}$.

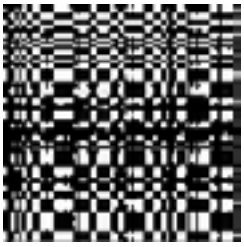

Figure 7. The reconstructed image $\mathbf{I}^{20}_{100x100}$.

Figure 8. The reconstructed image $\mathbf{I}^{30}_{100x100}$.

Figure 9. The reconstructed image $\mathbf{I}^{50}_{100x100}$.

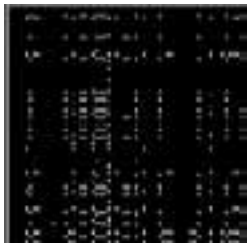

Figure 10. The reconstructed image $\mathbf{I}^{70}_{100x100}$.

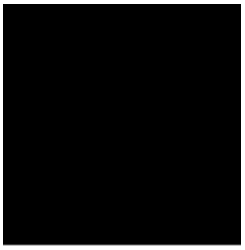

Figure 11. The reconstructed image $\mathbf{I}^{100}_{100x100}$.

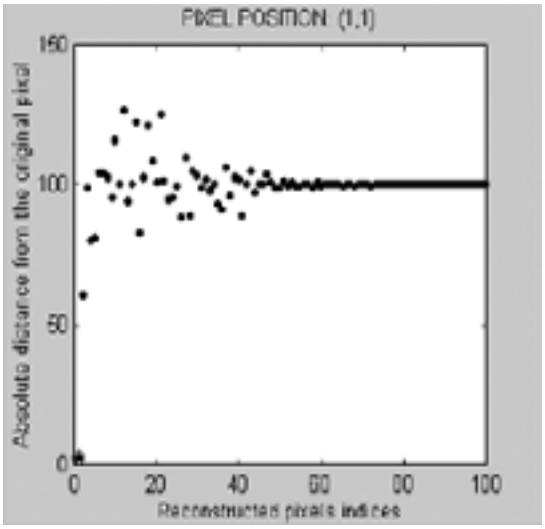

Figure 12. Reconstruct images distances to pixel (1,1).

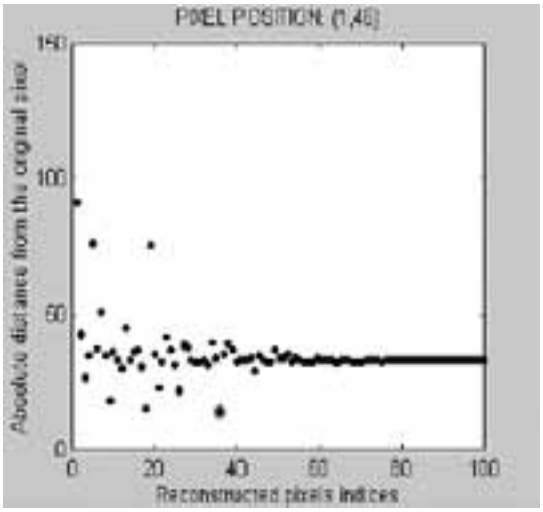

Figure 13. Reconstruct images distances to pixel (1,46).

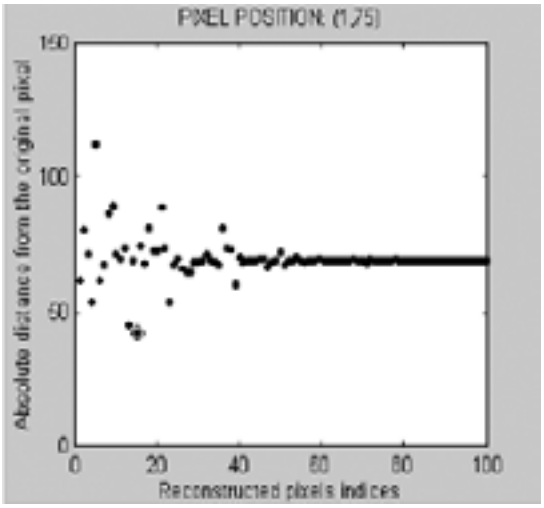

Figure 14. Reconstruct images distances to pixel (1,75).

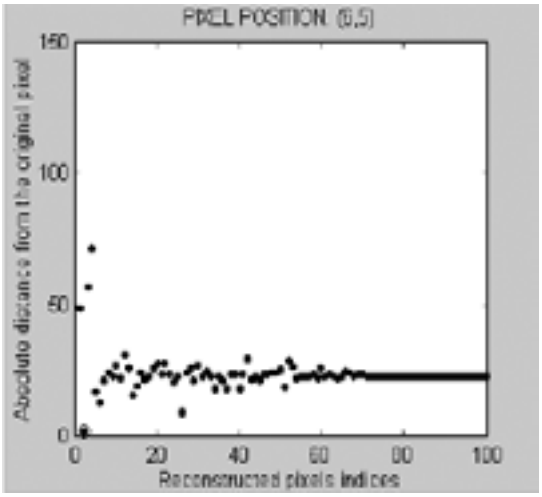

Figure 15. Reconstruct images distances to pixel (6,5).

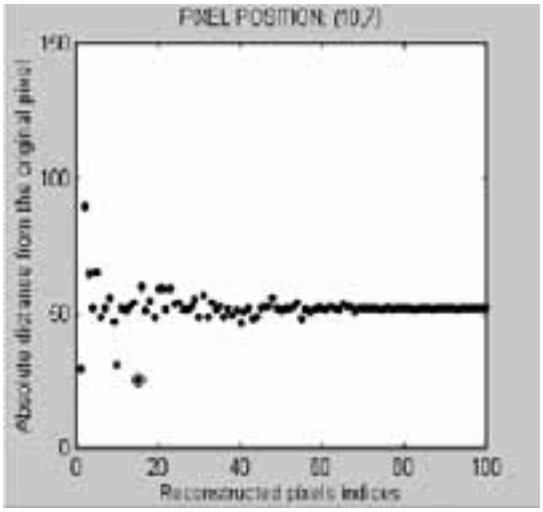

Figure 16. Reconstruct images distances to pixel (10,7).

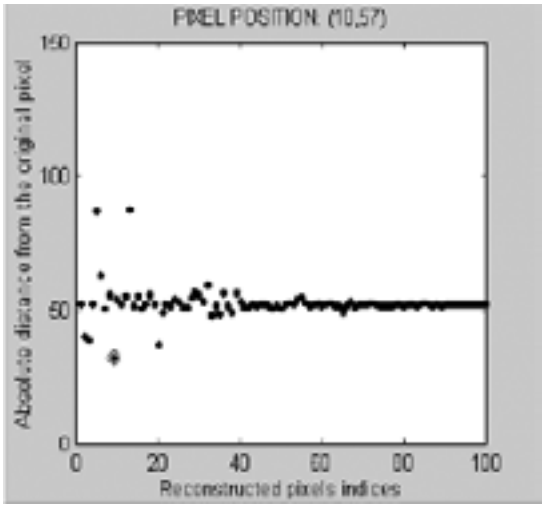

Figure 17. Reconstruct images distances to pixel (10,57).

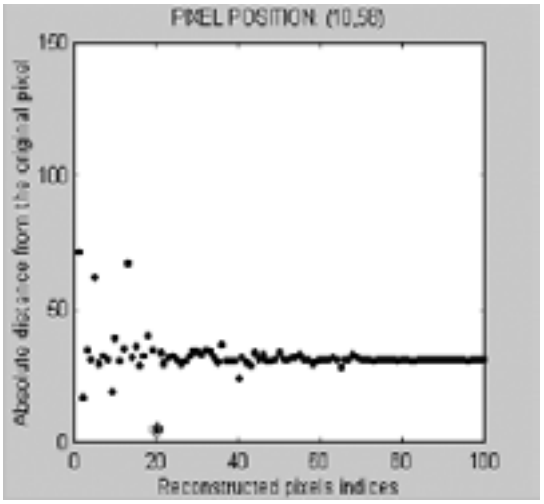

Figure 18. Reconstruct images distances to pixel (10,58).

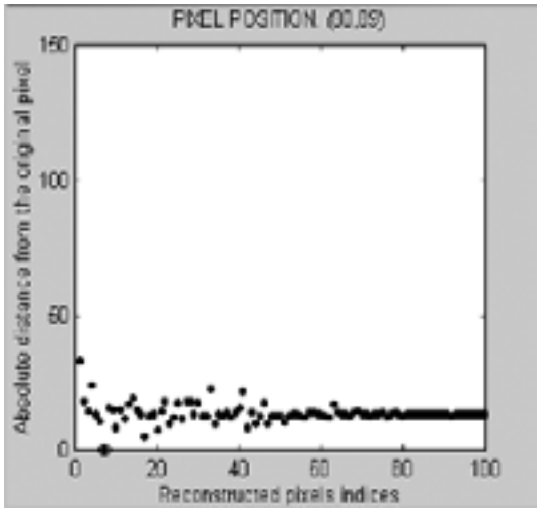

Figure 19. Reconstruct images distances to pixel (80,89).

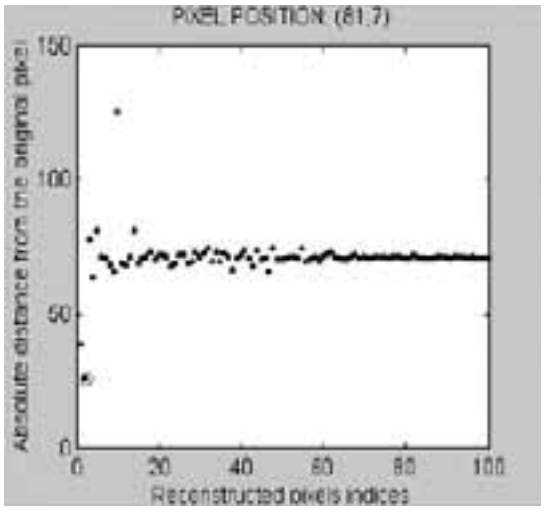

Figure 20. Reconstruct images distances to pixel (81,7).

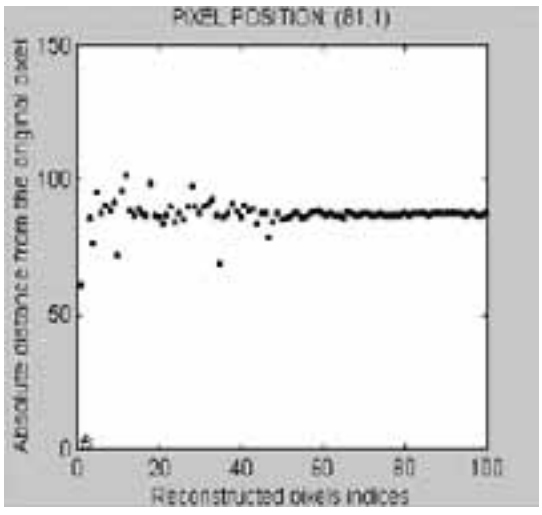

Figure 21. Reconstruct images distances to pixel (81,1).

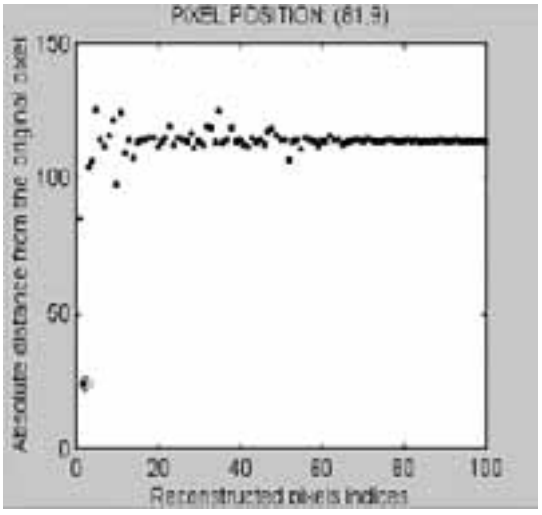

Figure 22. Reconstruct images distances to pixel (81,9).

4. Unsupervised classification method

An important feature of neural networks is the ability to *learn* from their environment, and through learning to *improve* performance in some sense. We can classify it in the supervised and unsupervised learning methods. In the supervised method the targets may take the form of a desired input-output mapping that the algorithm is required to approximate. In another way, the purpose of an unsupervised learning or self-organized learning is to *discover* significant patterns or features in the input data, and to do the discovery *without a teacher*. Doing so, the algorithm is provided with a set of rules of a *local* nature, which enables it to learn to compute an input-output mapping with specific desirable properties; the term "local" means that the change applied to the synaptic weight of a neuron is confined to the immediate neighborhood of that neuron. It frequently happens that the desirable properties represent goals of a neurobiological origin. Indeed, the modeling of network structures used for self-organized learning tends to follow neurobiological structures to a much greater extent than is the case for supervised learning.

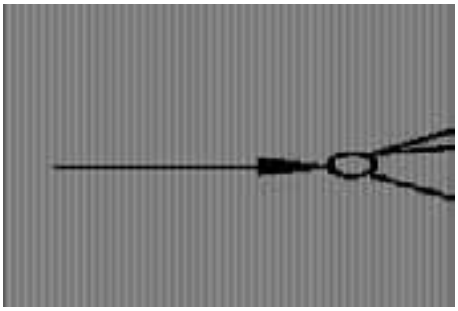

Figure 23 Structure of the neural classifier.

The structure of a self organizing system may take on a variety of different forms. It may, for example, consist of an *input* (source) layer and an output (representation) layer, with feedforward connections from input to output and lateral connections between neurons in the output layer. Another example is a feedforward network with multiple layers, in which the self-organization proceeds on a layer-by-layer basis. In both examples, the learning process consists of repeatedly modifying the synaptic weights of all the connections in the system in response to input (activation) patterns and in accordance with prescribed rules, until a final configuration develops [7].

In order to verify the potentiality of the denoise method on get a better image to classify comparing it with the original data, the unsupervised neural classifier is a good choice, since it doesn't need of any interference in its learning process. It was used the same structure and learning algorithm to map two structures using, respectively, the original and filtered images. In this way it is guaranteed the same neighborhood function, learning ratio and convergence space to both of the training set.

5. The classifier structure and the learning algorithm

The classifiers structure has two layers (Fig.23). The first one with one neuron and the second with fifty neurons, i.e., the training set has a convergence space of fifty classes. The local variables of the initial configuration of the training phase were fixed in a learning rate of 0.8 and the neighborhood space in the total of the space of convergence [8].

After 1000 training cycles, the net trained with the original image data had a final map with 18 distinct clusters. The net that was trained with the filtered image data had too a final map with 18 distinct clusters (Fig. 24).

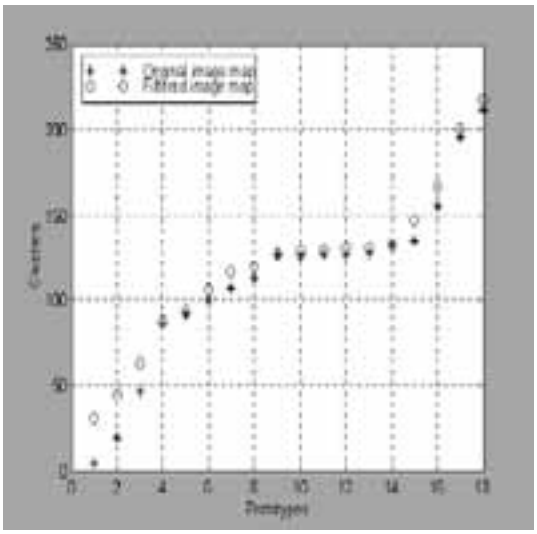

Figure 24. Clusters detected in the original and filtered images.

6. Analysis of the filtering and classifications

The fact of both final nets configurations have the same number of detected clusters and that they are very close (Fig. 24) is important to demonstrate that the filtered image kept the same features of the original data (Fig, 25). Some small, but important differences between the two maps are in the regions of the low value clusters. The firsts clusters of the net trained with the original data have small and close values while the filtered image map has its first clusters with higher values. These clusters classify the dark regions of the image. This feature demonstrates the effect of the filtering in the dark regions without degradation of the others ones (Fig. 22).

Figure 25. Filtered Image.

In order to have a visualization of the effects of the two maps generated (Fig. 24), the original data was classified with these two different clusters sets (Fig. 29), getting the classification A from the map one (Fig. 26) and the classification B from the map two (Fig. 27).

All of these algorithms were implemented in the Matlab 7.0 environment using a personal computer 512 Mhz with 1024 Mb RAM.

In the case of this scene, the black region is a lake and the light pixels are construction regions (Fig. 27). The classified images are more interesting to show visually the differences of the clusters of the two nets. The classification of the lake region of the net trained with the filtered image (Fig. 27) is more homogeneous than the classification of the net trained with the original image (Fig. 26).

Figure 26. Classification A (see text).

Figure 27. Classification B (see text).

7. Conclusions and future works

The utilization of Self Organizing Map was useful to demonstrates the filter effects. Due its unsupervised classifier feature, there wasn't interference in the process of clusters recognition of the original and filtered images (Fig. 28). So, it was possible to compare the classifications of the original and the filtered images.

The study of the features of images generated by the reconstruction process from part of the single values matrix demonstrated the effect of each single value in the formation of each image pixel.

Changes in the single values could have many effects in the reconstructed image. But, special forms of changes in the single values can arise in interesting results in the reconstructed image.

At this work the objective of maximizing the signal-to-noise ratio was considered to define a criteria of choice of the single values to reconstruct the filtered image. It was possible, by the analysis of the reconstructed images of each single value, to verify what single value had more influence to each pixel of the original data. So, with this criteria, the final reconstruct image pixels have the influence of the single values, that are not its fundamental single value, minimized.

As consequence of it, the clusters detected by the Self Organizing Map Net trained with the filtered data didn't detect the small variation in the regions of the digital image where the signal-to-noise ratio were down, i.e., potentiality noises in these regions didn't interfere in the homogeneous characterization of their class.

An immediate development of these results is to study how to combine many "fundamental influences" of single values (taking these from the images reconstructed from a unique single value presented here) of a same pixel to take its reconstruction. This would arise in a finely way of to reconstruct better approximations of the original data

excluding the variation regions where there's noise. The goal here is to get different degree of finely in each reconstructed pixel.

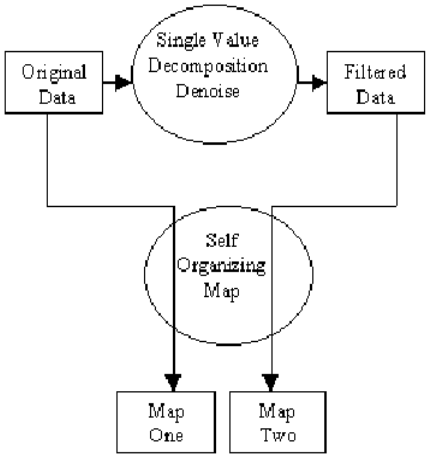

Figure 28. Diagram of block 1.

The classification experiments of this work was fixed in one structure of neural net, with the same local configuration of the learning algorithm to improve a same environment of convergence. But, it would be interesting to verify another configuration and structures of neural classifiers and another kinds of unsupervised classifiers in order the take more sensibility on the clusters detections.

The conjugate use of a more sensible classifier with the combination of more than one reconstructed pixels will improve more finely in the filtering and classifying sequentially processes.

Another way is to think about all the reconstructed pixels as input vectors to self organizing map. In this way the neural net will create another space of clusters to classify an image.

This study search increase the utilization field in the Recognition and Facial Identification. This biometric with technical that associate research in great facial database, they are going to enable people's utilization fast demanded and pendent with federal justice at Airports, Border Harbors and Zones.

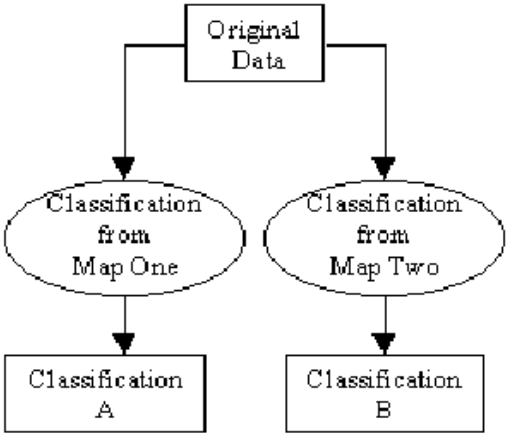

Figure 29. Diagram of block 2.

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